PIPE DISCHARGE FLOW CALCULATIONS
(A DIERS Users Group Round-Robin Exercise)

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Current Pipe Flow Benchmark

- Pipe discharge mass flux calculations
- HEM (homogeneous-equilibrium model) assumption.
- Same 3 systems, 4 inlet qualities (incl. vapor)

  $x_o = 0.0001, 0.01, 0.1, 1.0$
- 2 pipe configurations -

  $N = K_{en} + 4f_F\ L/D = 1.5$ (L/D = 50)

  $N = K_{en} + 4f_F\ L/D = 5$ (L/D = 225)

  fully turbulent two-phase, $f_F = 0.005$ (Fanning)

  sharp-edge entrance, $K_{en} = 0.5$

Horizontal Pipe Discharge Problem

Same identical inlet (two-phase) conditions as the nozzle case.

- Two different piping (frictional) resistance

	Pipe I	**Pipe II**
D	2.067 in	2.067 in
L/D	50	225
L	8.61 ft	38.8 ft
K_{en}	0.5	0.5
N	1.5	5.0

Note - $N = K_{en} + 4f_{TP}\ L/D$, with $f_{TP} = 0.005$, and $K_{en} = 0.5$

cyclo-Hexane saturated vapor decompression (PR Eos) – ck to see if Xv < 1.0 (condensation)

P	TC	SL	SV	S mix	Xv	VL	VV	V 2ph
bar a	C	J/moleK	J/moleK	J/moleK	-	m3/kmol	m3/kmol	m3/kmol
10	182.28	-15.08	35.445	35.445	1.0000	0.136	3.069	3.069
9	176.25	-18.03	34.365	35.445	1.0206	0.1337	3.424	3.49184
8	169.69	-21.23	33.193	35.445	1.0414	0.1313	3.863	4.01741
7	162.49	-24.73	31.919	35.445	1.0622	0.1289	4.422	4.68923
6	154.46	-28.61	30.514	35.445	1.0834	0.1264	5.157	5.57653
5	145.36	-32.99	28.956	35.445	1.1047	0.1239	6.171	6.80443
4	134.76	-38.08	27.194	35.445	1.1264	0.1212	7.665	8.61859
3	121.91	-44.22	25.156	35.445	1.1483	0.1183	10.102	11.5826
2	105.21	-52.18	22.707	35.445	1.1701	0.1149	14.832	17.3352
1	79.96	-64.21	19.564	35.445	1.1896	0.1107	28.372	33.7295

Summary of Isothermal FLASH for Benchmark Two-component Systems

P (bar)	TC	Xi C2	Yi C2	MW L	MW V	L/F mole	Xv mass
BAR	C	mole frac	mole frac	kg/mole	kg/mole	-	-
10	51.92	0.200	0.9752	86.18	31.806	0.99988	0.0001
10	51.92	0.200	0.9752	86.18	31.806	0.97357	0.0099
10	51.92	0.200	0.9752	86.18	31.806	0.76863	0.1000
P (bar)	TC	Xi N2	Yi N2	MW L	MW V	L/F mole	Xv mass
BAR	C	mole frac	mole frac	kg/mole	kg/mole	-	-
33	25	0.025	0.9936	82.756	28.36	1.00000	0.0000
33	25	0.025	0.9936	82.756	28.36	0.97108	0.0101
33	25	0.025	0.9936	82.756	28.36	0.75531	0.0999

C2-C7 isothermal FLASH (10 bar,51.9C) xo=0.01

- COMP 1= C2H6, Comp 2= C7H16
- X0(1),X0(2),MWF= 0.2204000 0.7796000 84.74335
-
- ISOTHERMAL FLASH (converged) RESULTS:
- P(input) = 10.00000 bar
- T(input) = 325.0600 K 51.89999 C
- X(1),X(2) = 0.1999601 0.8000399
- Y(1),Y(2) = 0.9752654 2.4734735E-02
- KBIN(1,2) = 0.01
- K(1),K(2) = 4.877294 3.0916872E-02
- ZL, ZV = 5.0781313E-02 0.9259123
- LL liquid frac= 0.9736363
- FUGL(1,2) = 9.132528 0.1732796 bar
- FUGV(1,2) = 9.132540 0.1732796 bar
-
- ZL,MWL= 5.0781313E-02 86.17680
- VL,HL,SL= 0.1372390 -2.5702150E+07 -70545.31
- ZV,MWV= 0.9259123 31.80465
- VV,HV,SV= 2.502323 964220.2 -14400.16
- ...LL.............VMIX.........HMIX/E3………..SMIX/E3...
- 0.97364 0.19959 -0.249991E+05 -69.06512

C2-C7 isothermal FLASH (10 bar,51.9C) xo=0.1

- COMP 1= C2H6, Comp 2= C7H16
- X0(1),X0(2),MWF= 0.3793000 0.6207000 73.59969
-
- ISOTHERMAL FLASH (converged) RESULTS:
- P(input) = 10.00000 bar
- T(input) = 325.0800 K 51.92001 C
- X(1),X(2) = 0.1999080 0.8000939
- Y(1),Y(2) = 0.9752389 2.4754573E-02
- KBIN(1,2) = 0.01
- K(1),K(2) = 4.878444 3.0939478E-02
- ZL, ZV = 5.0781257E-02 0.9259216
- LL liquid frac= 0.7686253
- FUGL(1,2) = 9.132414 0.1734275 bar
- FUGV(1,2) = 9.132404 0.1734281 bar
-
- ZL,MWL= 5.0781257E-02 86.18064
- VL,HL,SL= 0.1372473 -2.5699248E+07 -70534.52
- ZV,MWV= 0.9259216 31.80584
- VV,HV,SV= 2.502502 965489.1 -14395.55
- ...LL.............VMIX........HMIX/E3………….SMIX/E3...
- 0.76863 0.68451 -0.195297E+05 -57.54538

Pipe Flow Formulation

- Constant diameter pipe (continuity) –

  $G = \rho u = \text{constant}$

- Energy balance (adiabatic flow) -

$$H_1 + \frac{1}{2} G_1^2 v_1^2 = H_2 + \frac{1}{2} G_2^2 v_2^2 = \text{constant}$$

- Momentum balance (turbulent flow) -

$$vdP + G^2 vdv + \frac{4f}{2D} G^2 v^2 dZ = 0$$

Expansion Law (Eq. of State)

- Need P-v (pressure - sp. volume) relation.
- Normal practice is to use constant H (enthalpy) flash calculation.
- From adiabatic flow starting from stagnation -

$$H_o = H + \frac{1}{2} G^2 v^2$$

a constant H flash assumes K.E. to be small.

General Differential Momentum Balance for Pipe Flow

$$dP + G^2 dv + \frac{1}{2} G^2 v \frac{4f}{D} dZ + \frac{g\cos\theta}{v} dZ = 0$$

Rearranging and moving dZ terms to L.H.S.

$$\frac{G^2 v^2}{v} 4f \frac{dZ}{D} + \frac{g\cos\theta L}{4fL/D} 4f \frac{dZ}{D} = -\, vdP - G^2 v\, dv$$

General integral momentum equation can be written as

$$\int \frac{4f}{D} dZ = -\int \frac{vdP + G^2 v\, dv}{\frac{G^2 v^2}{2} + \frac{g\cos\theta L}{4fL/D}}$$

L.H.S. with f constant becomes $4fL/D = N$, total piping resistance.

Horizontal Pipe Discharge for HEM

Here cosθ = 0, gLcosθ drops out, it is more convenient to rewrite the integral momentum equation explicitly in G as

$$G^2 = \frac{-\int_{P_1}^{P_2} \frac{dP}{v}}{\int_{v_1}^{v_2} \frac{dv}{v} + \frac{1}{2} N}$$

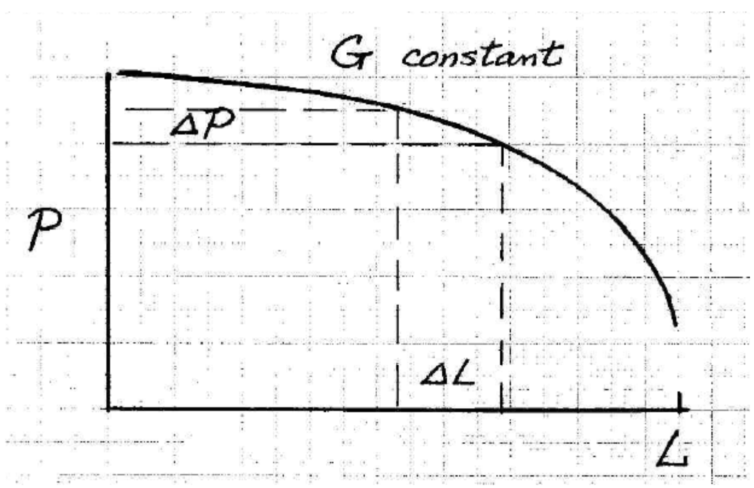
$$G^2 = \frac{2\int_{P_2}^{P_1} \frac{dP}{v}}{2 \ln \frac{v_2}{v_1} + N}$$

Thus a P-v expansion law would allow analytical/numerical integration.

Pipe-Segment Numerical Integration

$$\Delta L = -\frac{\bar{v}\,\Delta P + G^2\,\bar{v}\,\Delta v}{\frac{2f}{D}\,G^2\,\bar{v}^2}$$

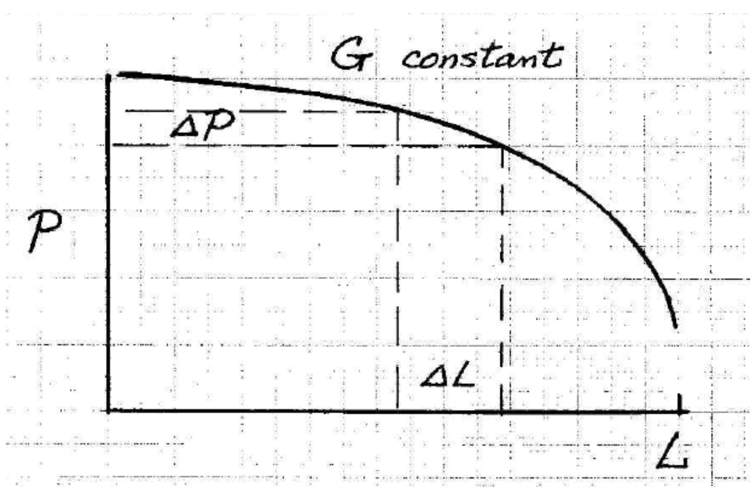
where ΔP is pressure increment
Δv is incremental specific volume over ΔP
$\bar{v}$ is average specific volume in ΔP



Pipe-Flow Computation coupled with FLASH

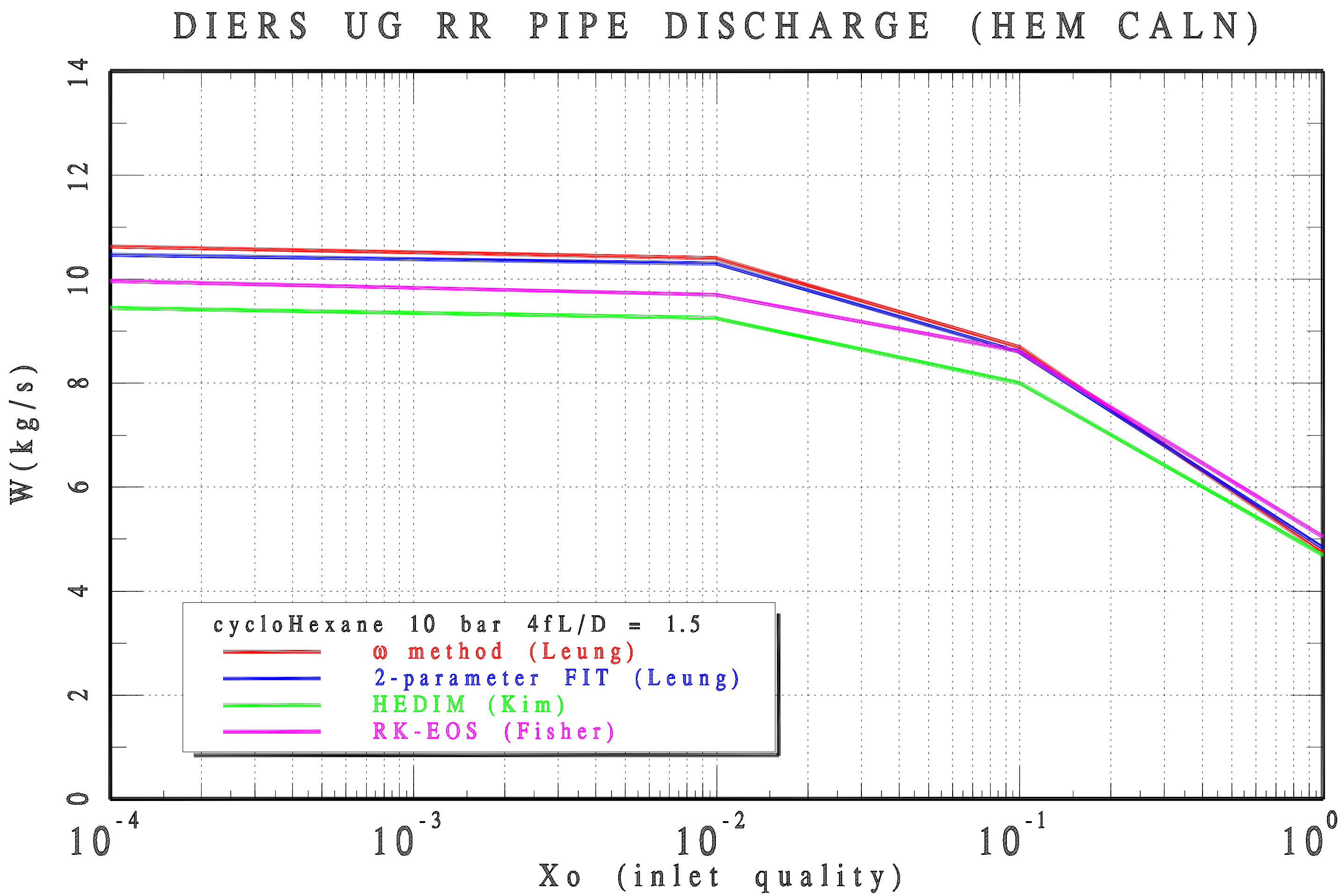
where Flash would not be constant H but rather obeying the energy eq. below

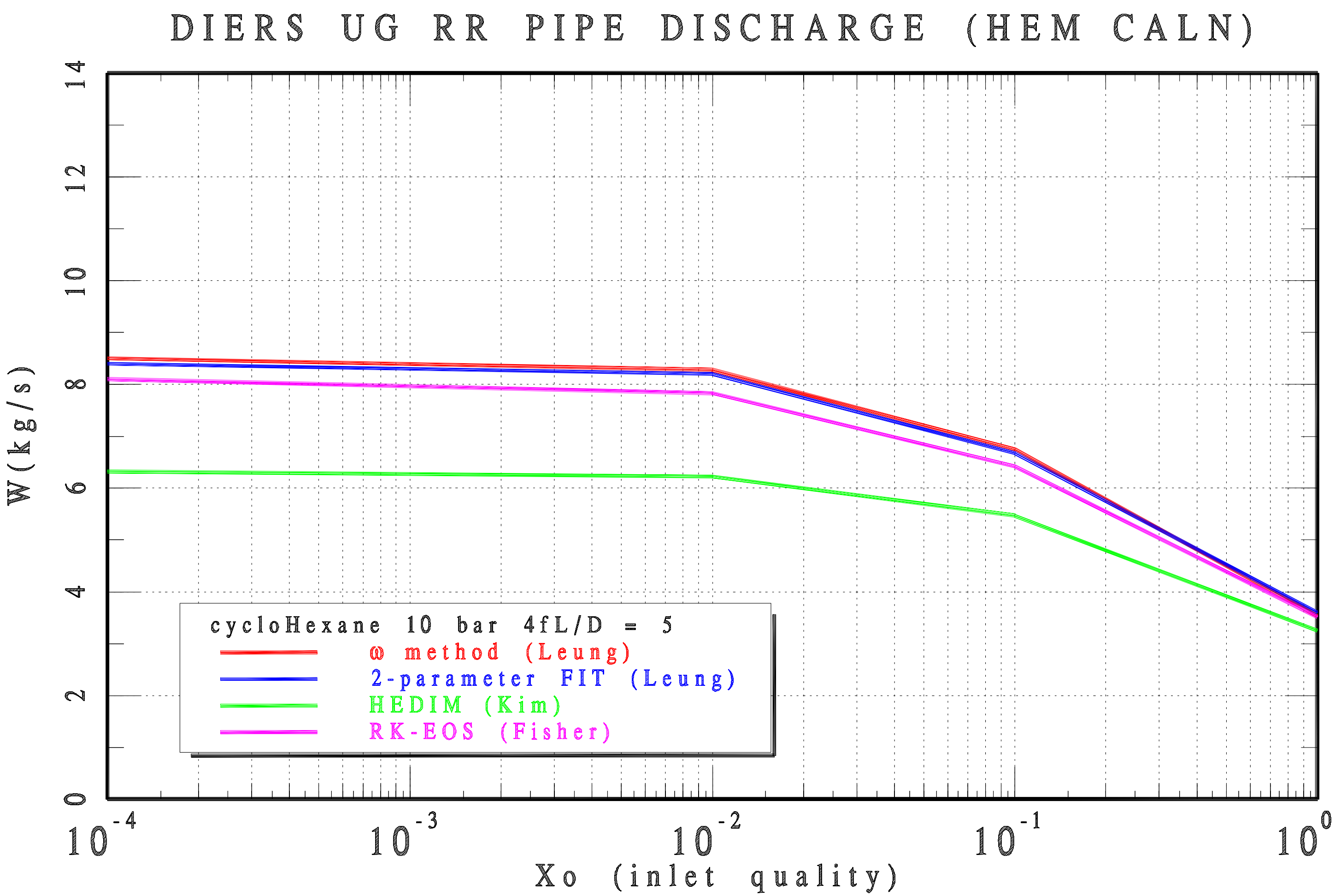
$$H_o = H + \frac{1}{2} G^2 v^2$$



cycloHEX pipe flow comparison (HEM model calculations)

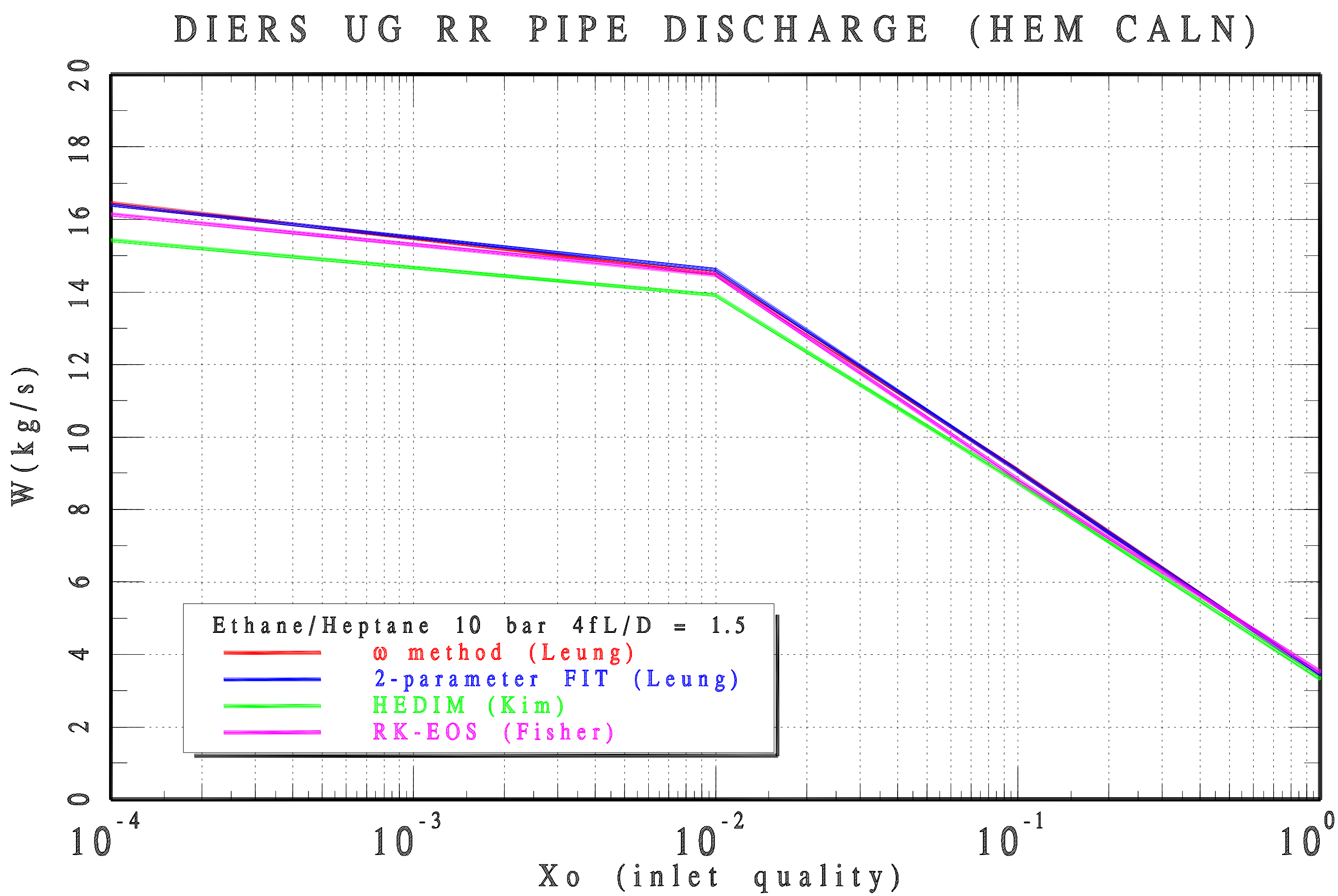
Case	**1a**		**1b**		**1c**		**1d**	
	Xo=0.0001		Xo=0.01		Xo=0.1		Xo=1.0	
Po (bar)	10	10	10	10	10	10	10	10
Rho (kg/m3)	603	603	509	509	196	196	27.4	27.4
Ap(in2)	3.355	3.355	3.355	3.355	3.355	3.355	3.355	3.355
4fL/D	**1.5**	**5**	**1.5**	**5**	**1.5**	**5**	**1.5**	**5**
Omega (Leung)								
W (kg/s)	10.63	8.50	10.40	8.28	8.69	6.75	4.74	3.53
2-par fit (Leung)								
W (kg/s)	10.47	8.40	10.30	8.20	8.60	6.67	4.83	3.6
Kim (Bayer)								
W (kg/s)	9.45	6.32	9.25	6.22	8.00	5.47	4.69	3.25
HG Fisher								
W (kg/s)	9.97	8.1	9.7	7.83	8.61	6.42	5.04	3.71

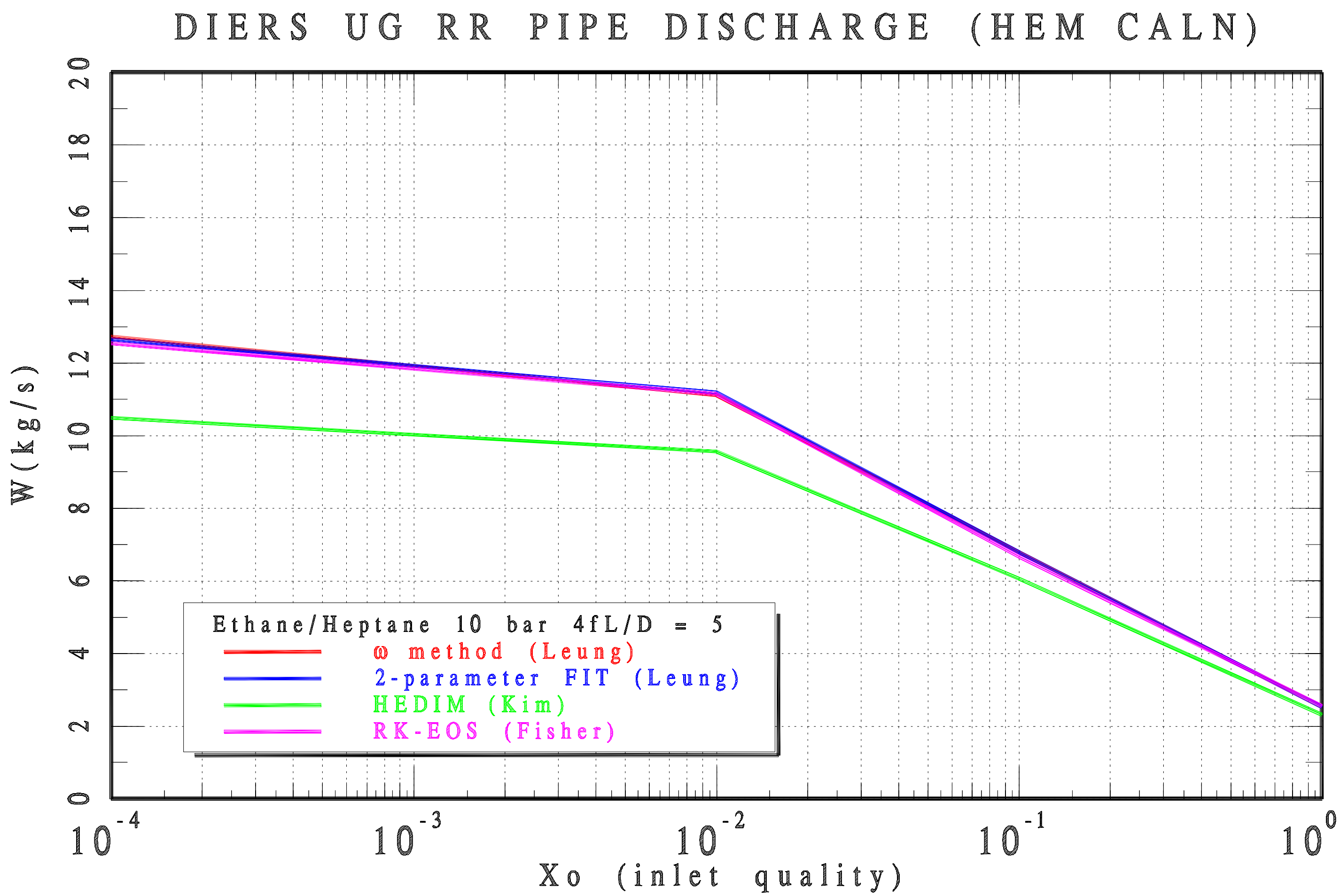

DIERS UG RR PIPE DISCHARGE (HEM CALN)
W (kg/s)
0
2
4
6
8
10
12
14
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
Xo (inlet quality)
cycloHexane 10 bar 4fL/D = 1.5
ω method (Leung)
2-parameter FIT (Leung)
HEDIM (Kim)
RK-EOS (Fisher)


DIERS UG RR PIPE DISCHARGE (HEM CALN)
W (kg/s)
0
2
4
6
8
10
12
14
cycloHexane 10 bar 4fL/D = 5
ω method (Leung)
2-parameter FIT (Leung)
HEDIM (Kim)
RK-EOS (Fisher)
10^-4
10^-3
10^-2
10^-1
10^0
Xo (inlet quality)

C2-C7 pipe flow comparison (HEM model calculations)

Case	**2a**		**2b**		**2c**		**2d**	
	Xo=0.0001		Xo=0.01		Xo=0.1		Xo=1.0	
Po (bar)	10	10	10	10	10	10	10	10
Rho (kg/m3)	626	626	425	425	107.5	107.5	12.71	12.71
Ap(in2)	3.355	3.355	3.355	3.355	3.355	3.355	3.355	3.355
4fL/D	**1.5**	**5**	**1.5**	**5**	**1.5**	**5**	**1.5**	**5**
Omega (Leung)								
W (kg/s)	16.45	12.72	14.50	11.11	9.08	6.79	3.42	2.53
2-par fit (Leung)								
W (kg/s)	16.40	12.65	14.61	11.19	9.06	6.78	3.42	2.53
Kim (Bayer)								
W (kg/s)	15.43	10.49	13.91	9.56	8.74	6.06	3.32	2.31
HG Fisher								
W (kg/s)	16.14	12.54	14.47	11.14	8.82	6.66	3.51	2.56

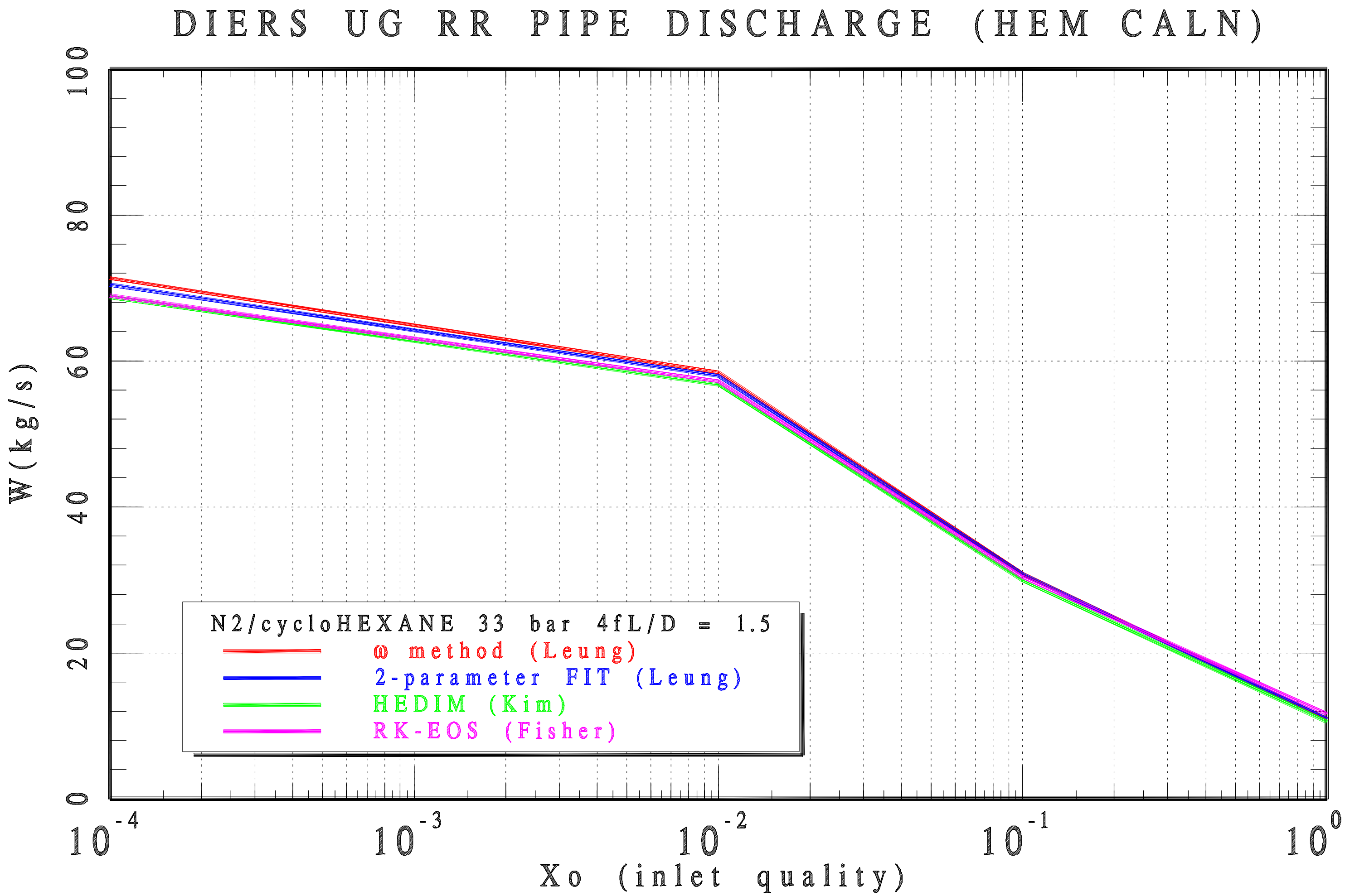

DIERS UG RR PIPE DISCHARGE (HEM CALN)
W(kg/s)
0
2
4
6
8
10
12
14
16
18
20
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
Xo (inlet quality)
Ethane/Heptane 10 bar 4fL/D = 1.5
ω method (Leung)
2-parameter FIT (Leung)
HEDIM (Kim)
RK-EOS (Fisher)

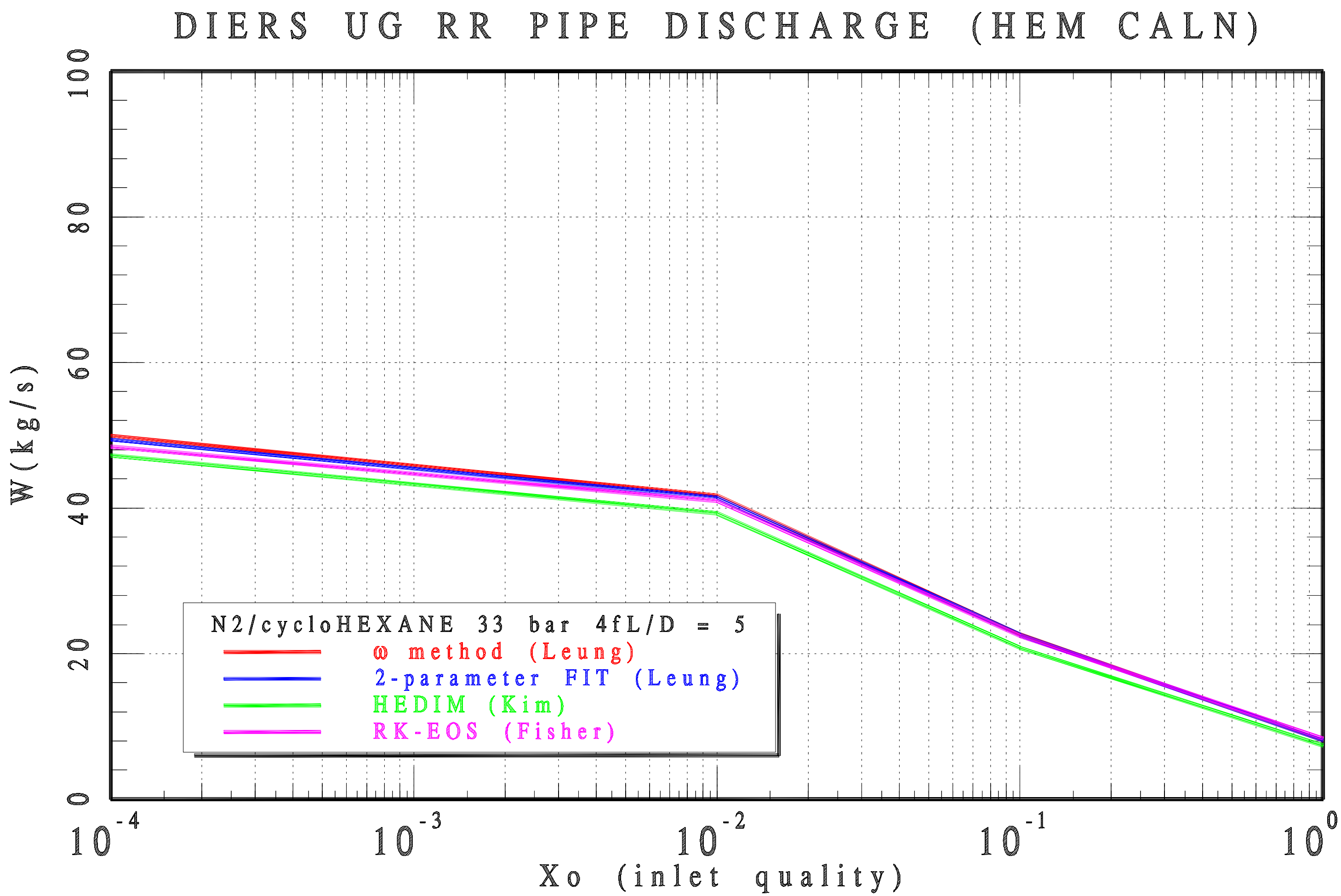


SOURCE GCL FILE IS F:\DIERS UG\DIERS Pipe flow Benchmark\Leung benchmk\C2-C7 comp plot.GCL

N2-cycloHEX pipe flow comparison (HEM model calculations)

Case	3a		3b		3c		3d	
	Xo=0.0001		Xo=0.01		Xo=0.1		Xo=1.0	
Po (bar)	33	33	33	33	33	33	33	33
Rho (kg/m3)	812	812	674	674	269	269	38.3	38.3
Ap(in2)	3.355	3.355	3.355	3.355	3.355	3.355	3.355	3.355
4fL/D	**1.5**	**5**	**1.5**	**5**	**1.5**	**5**	**1.5**	**5**
omega (Leung)								
W (kg/s)	71.37	49.97	58.44	41.73	30.83	22.64	10.97	8.09
2-par fit (Leung)								
W (kg/s)	70.48	49.41	57.98	41.45	30.79	22.58	10.95	8.09
Kim (Bayer)								
W (kg/s)	68.76	47.24	56.80	39.29	29.94	20.82	10.57	7.35
HG Fisher								
W (kg/s)	68.96	48.44	57.24	40.96	30.41	22.44	11.61	8.34


DIERS UG RR PIPE DISCHARGE (HEM CALN)
W (kg/s)
0
20
40
60
80
100
10^-4
10^-3
10^-2
10^-1
10^0
Xo (inlet quality)
N2/cycloHEXANE 33 bar 4fL/D = 1.5
ω method (Leung)
2-parameter FIT (Leung)
HEDIM (Kim)
RK-EOS (Fisher)


DIERS UG RR PIPE DISCHARGE (HEM CALN)
W(kg/s)
0
20
40
60
80
100
10^-4
10^-3
10^-2
10^-1
10^0
Xo (inlet quality)
N2/cycloHEXANE 33 bar 4fL/D = 5
ω method (Leung)
2-parameter FIT (Leung)
HEDIM (Kim)
RK-EOS (Fisher)

Final Call for Participation

- In-house program for pipe flow
- SIMSCI module for two-phase flow
- SuperChem
- SuperChem DIERS Lite
- ASPEN
- Submit to Joe Leung (design/testing chair) @leunginc @cox.net or Harold Fisher before next DIERS UG mtg